A STUDY ON STATIC GAITS FOR A FOUR LEG ROBOT

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ABSTRACT

This paper describes the study of algorithms to obtain static or marginally static gaits. Some of them are similar to the walking gaits used by some mammals, e.g. horses, cows, etc.. It describes a methodology for deriving the inverse kinematics of a quadruped. It then explains an algorithm which implements an exhaustive search of static gaits using a formal description of the robot. A similar method is applied to implement on a quadruped robot the most common walking gaits seen in animals. 1625 stable and 400 marginally stable robot configurations were found using our stability test algorithm. Using these results in the first algorithm, 848 static gaits and 141424 marginally static gaits were found. By applying the second algorithm were found 15 walking gaits and were analysed to determine their stability margins (which is the minor of the minimum distance of the mass centre to the border of the supporting polygon formed by the legs in contact with the ground). The method can be used to determine similar gaits on walking systems with any number of legs.

1. INTRODUCTION

The research project that this paper describes is the ALEGRO project, whose name stands for A LEGged RObot - ALEGRO, was initiated in October of 1998 at the Institute of Systems and Robotics of the University of Coimbra. It started with a exoskeleton of a quadruped - TITAN VIII [4]. Its main aim is to build an autonomous system able to navigate in natural terrain.

In order to reduce the volume and the space occupied for the control system we opted for a industrial computer based on a Pentium processor, with a PC/104 bus (which serves the purpose of interfacing with DAC and ADC boards) and several others types of interfaces, which allow us to connect many sorts of peripherals in a small motherboard.

For the direct kinematics problem we used the Denavit-Hartenberg model. For the inverse kinematics study we developed a new methodology named Trigonometric Relations Method that we will describe later and which was applied to our system.

Our research focused on static gaits and two approaches were used: first we developed an exaustive search of static gaits using a formal description of the robot and imposing some strong restrictions. Secondly, we used a similar formal method to implement the most common gait in animals - the walking gait used by horses, dogs, cows and almost all the quadrupeds.

Using the first approach 848 static gaits and 141424 marginally static gaits were found. For the second approach were found 15 walking gaits that were analysed to determine their stability margins. Although our specific study was made in quadrupeds, our methods can be generalised to any number of legs.

In the next section we present the inverse kinematics of the ALEGRO. In the section 4 it is described the study on static gaits and in the last section we present some future work.

2. INVERSE KINEMATICS

The legs of the robots can be considered manipulators so all manipulator's math models can be applied [3].

To solve the direct kinematics problem we used the model of Denavit-Hartenberg [3].

The inverse kinematics is in general more complex than the direct one, due to the fact that there isn't only one solution and in some cases there is an infinity number of solutions. To solve this problem the Jacobian's Math Method is the most used. In the case of our quadruped, the number of solutions is just one due to physical restrictions and then we developed a simple method (Trignometric Relations Method) to legs (also appliable to manipulators) with three d.o.f..

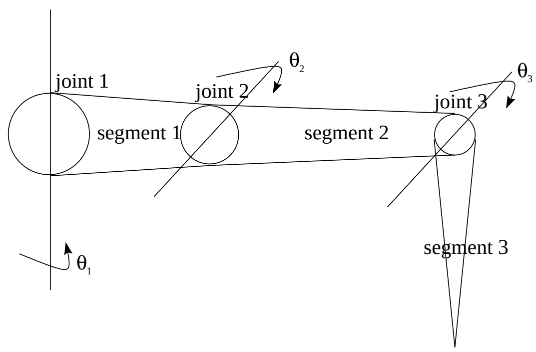

Figure 1: The Leg with three d.o.f.

Our known variables are the coordinates of the extremity of leg - (x_f, y_f, z_f) - in the base leg coordinate system and the length of the segments (a_1, a_2 and a_3) and we expect to calculate the angles θ_i (i=1,2,3) for each joint.

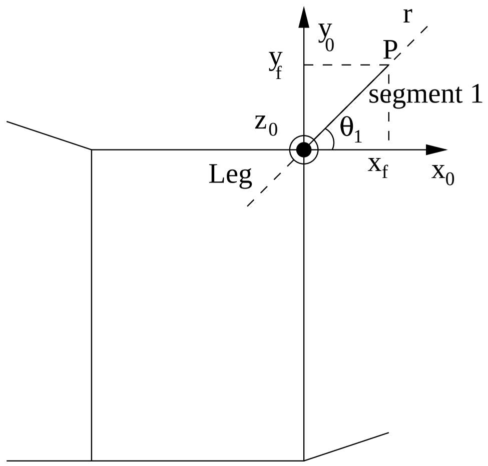

Figure 2: Calculation of θ_1

The first d.o.f. of the leg (joint 1) permits the leg to rotate about the vertical axis, so θ_1, as can be seen on figure 2, is the angle between segment 1 and the xx axis, and is given by:

$$\theta_1 = arctan\left(\frac{y_f}{x_f}\right) \tag{1}$$

The calculation of the angles θ_2 and θ_3 is made simultaneously because they are associated with parallel joints. We can see in figure 3 that the segments 2 and 3 form a triangle. With the coordinates of two points (A and P) and the length of the segments we calculate those angles.

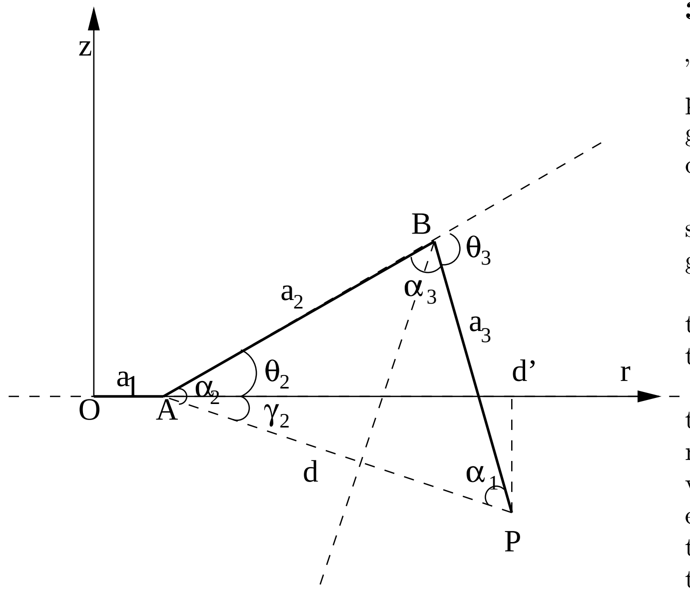

Figure 3: Calculation of θ_2 and θ_3

From the figure 3 we can derive some expressions:

$$\begin{aligned}
A &\equiv (a_1\cos(\theta_1), a_1\sin(\theta_1), 0) \\
d &= distance(A, P) = \\
&= \sqrt{(a_1\cos(\theta_1) - x_f)^2 + (a_1\sin(\theta_1) - y_f)^2 + z_f^2} \\
d' &= \sqrt{x_f^2 + y_f^2} - a_1 \\
\alpha_i &- \textit{internal angles of the triangle } [ABP] \\
&\quad (i = 1, ..., 3) \\
\gamma_2 &= \arccos(\frac{d'}{d})
\end{aligned}$$

Now that the variables and constants are defined, we can use simple mathematics to calculate the values of the angles that are:

$$\begin{aligned}
\theta_2 &= \gamma_2 - \alpha_2 \\
\theta_3 &= \pi - \alpha_3 = \alpha_1 + \alpha_2
\end{aligned}$$

$$\begin{aligned}
\theta_2 &= \arccos\left[\frac{a_2^2 - a_3^2 + t}{2a_2\sqrt{t}}\right] - \\
&- \arccos\left[\frac{\sqrt{x_f^2 + y_f^2} - a_1}{\sqrt{t}}\right]
\end{aligned} \tag{2}$$

$$\begin{aligned}
\theta_3 &= \arccos\left[\frac{a_2^2 - a_3^2 + t}{2a_2\sqrt{t}}\right] + \\
&+ \arcsin\left[\frac{a_2}{a_3}\sin\left(\arccos\left[\frac{a_2^2 - a_3^2 + t}{2a_2\sqrt{t}}\right]\right)\right]
\end{aligned} \tag{3}$$

with

$$t = (a_1\cos(\theta_1 - x_f))^2 + (a_1\sin(\theta_1 - y_f))^2 + {z_f}^2$$

3. STATIC GAITS

"Gait is the leg phasing part of the coordination problem." [5]. In other words, we can say that, a gait defines the form and the characteristics of a body displacement.

The gaits can be divided in three basic types: static gaits, marginally static gaits and dynamic gaits.

In a quadruped robot three of the legs must be on the ground for a static gait. So this constraint limits the number and type of gaits that can be used.

Dynamic gaits, on the other side, don't have this limitation because, as long as equilibrium is maintained, the number of legs on the ground can vary from 0 during a jump to the total number of existing legs. The problem that arises in this cases is the need to a complete dynamic model of the robot that can be very complex.

Our research focused on static gaits and two approaches were used. As a first approach for the static gait analysis we used a description of the robot that we named as 2D+1 model. Using this model and imposing some strong restrictions, we developped an exaustive search of static gaits.

In the second approach we studied the walking gait. We used a similar method to implement the most common gait in animals, the walking gait, which is used by horses, dogs, cows and almost all the quadrupeds.

In the next subsection we'll explain the formal method used and subsequently we'll explain both approaches to static gaits.

3.1. 2D+1 Model

For the static gait analysis we created a simple model that describes the position of the leg tips with respect to the body and that was called 2D+1 model. As can be seen on figure 4, this model describes the position of the toe with respect with body and the binary state that can be raised on set on the ground.

Using this model there is a loss of information, because it does not integrate any information about the pose of the body or on how much the raised leg(s) is(are) raised.

The body of the robot is described by a polygon, the legs by straight lines and the extremity of the legs by empty (○) or full (●) circles depending on the state of the leg (raised or set on the ground).

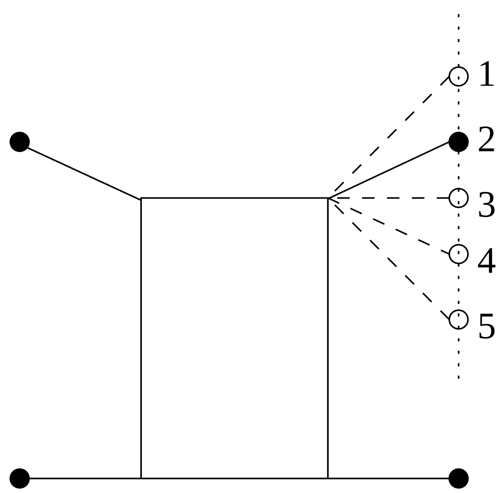

Figure 4: 2D+1 Model of a quadruped

The center of the polygon is the origin of the coordinate system and it is considered as the center of mass. This assumption is a simplification of the truth because in fact the center of mass is disturbed by the variation of the distribution of mass along the phase of the leg. However, we considered that this disturbance is minimal because the legs are much lighter than the body of the robot and for simplicity the center of the mass is considered always on the center of the body.

Another consideration is that each leg can only take discrete positions from a predefined set. The number of diferent positions in this set is one of the parameters of the model that can be adjusted. Each leg can then be in one of the five positions and it can be raised or set down on the ground (figure 4).

In the rest of this article, the configuration of leg j will be defined by $lc_j(i, s)$, where $i = 1, .., 5$ and $s = \{u, d\}$ (u - leg raised, d - leg on the ground). The robot configuration will be defined by the set of four leg configurations $\{lc_1, lc_2, lc_3, lc_4\}$.

3.2. Exaustive search of static gaits

Calculation of all the stable robot configurations

Using the 2D+1 model and the set of all possible configurations of the robot, we start to verify the stability of each robot configuration and eliminate those that fail this test.

The coordinates of the extremities of the legs are known so the coordinates of the vertices of the supporting polygon are also known.

The stability of one robot configuration is granted if the projection of the center of mass falls inside the polygon that has the supporting leg tips as vertices.

Obviously when all legs of the robot are set down on the ground, in any case the robot configuration is stable. Otherwise, when less than three legs are set down on the ground, the robot configuration is unstable or marginally stable. With three legs on the ground the robot configuration can be stable, marginally stable or unstable.

For a quadruped (figure 4) with ten possible positions of the legs (five of them raised and the other five on the ground) the number of possible robot configurations are $5^4 \times 2^4 = 10000$. After having applied this method the results obtained were:

- 1625 stable robot configurations;
- 400 marginal stable robot configurations;
- 7975 unstable robot configurations.

Algorithm to search static gaits

After the reduction of the number of possible robot configurations by eliminating all the unstable ones, we performed a search in the set formed by all stable and marginally stable robot configurations trying to obtain sequences of robot configurations that can be used as gaits.

The method uses several rules and restrictions to rearrange some sequences of robot configurations, in order to derivate static gaits. For our case study were used the following rules and restrictions:

- The maximum number N of robot configurations in a sequence is defined *a priori*;

- There are two types of movements:
  - to raise the leg or to set down the leg on the ground in one of the set of allowed positions;
  - to move the body forward, all legs do the same movement backward on the ground. This operation is called body displacement;
- At any time the maximum number of legs raised is one;
- The robot can be in one of the five states:
  - State 0 → There is one leg raised;
  - State 1 → All legs are set down on the ground; at least one leg didn't move yet; the body displacement still to be done;
  - State 2 → All legs are set down on the ground; all legs already move; the body displacement still to be done;
  - State 3 → All legs are set down on the ground; at least one leg didn't move yet; the body displacement has already been done;
  - State 4 → All legs are set down on the ground; all legs already move; the body displacement has already been done;
- A set of ordered robot configurations is a gait if the last robot configuration is the same as the first one and if the robot is in the state 4. This means that to consider a gait the robot must move all legs and make at least a body displacement;
- When a leg is raised the next robot configuration is a similar configuration but with all the legs set down on the ground;
- The number of legs that change their contact points with the soil from one robot configuration to the next can be 0, 1 or 4;
  - 0 → a raised leg set down on the ground;
  - 1 → a leg changed its leg configuration and raised;
  - 4 → the robot made a body displacemente;
- A leg can move in two consecutive robot configurations if one of them is a body displacement;
- A particular robot configuration can't be repeated in a gait.

Since the maximum number of robot configurations is N and the minimum number of robot configurations in a gait is 9 (=4 legs $\times$ 2 movements + 1 body displacement), due to the rules and restrictions imposed, the number of possible sequences of robot configurations that must be checked, for N=20, is

$$\sum_{n=9}^{20} \frac{1625!}{(1625-n)!} \approx 1.47 \times 10^{64} \qquad (4)$$

This number is very sensitive to the maximum number of robot configurations in a gait (N) so the rules and restrictions must be strong to avoid the combinatorial explosion.

Results

For a maximum number of robot configurations of twenty ($N = 20$) the results obtained were:

- 848 static gaits
- 141424 marginally static gaits

Some body displacement correspond to the transition of all the legs from a $lc_j(i, d)$ to a $lc_j(i + 1, d)$ and some others to a $lc_j(i + 2, d)$, these obviously faster than the first ones.

We applied the restrictions of a minimum of two positions body displacement and the number of gaits was reduced from 848 to 180.

Some of the obtainned gaits have been tested in our platform and the results were quite satistactory because although the gaits obtainned are not optimal all the tested ones would make the robot to move forward.

Conclusions

In this subsection we presented a method of analysis of stability and an algorithm of exaustive search for static gaits using a formal description of a robot.

The algorithms developped are simple but able to generate a great number of different static and marginally static gaits that conform to the restritions imposed.

Although the number of gaits obtained is high, specially for marginally static gaits, we observed a drastic reduction from the total number of sequences of robot configurations (that is the number of cases that the algorithm checked).

In addition, there are a lot of parameters that can be used to study with more detail some characteristics and that can be tuned in order to optimize the eficiency of the locomotion. Some of those parameters are:

- Maximum number of robot configurations;
- Number of legs;

- Number of allowable positions of legs;
- Minimum stability margin;
- Physical dimensions and geometry of the body and legs.

3.3. Walking Gait

The walking gait is settled by the definition of the trajectories of the robot legs (the same trajectory for all legs) and the phase displacement between them.

The aim is to derive a trajectory for one leg and replicate it in the remaining ones. As can be seen, all legs perform the same movement simultaneously although there is a fixed diference between the phase of each pair.

Developping a trajectory for the legs

The first step in developping the walking gait is to define the trajectory of legs.

To be possible the study of the temporization of the trajectory of one leg, it's necessary to define a path to be followed by the extremity of the leg, associated with a function of time.

The trajectory of the leg chosen is composed by two lines. The first line describes a circumference arc between two points on the ground and the second line is simply a straight line that join the two first points (the points on the ground). The trajectory chosen can be seen in the figure 5.

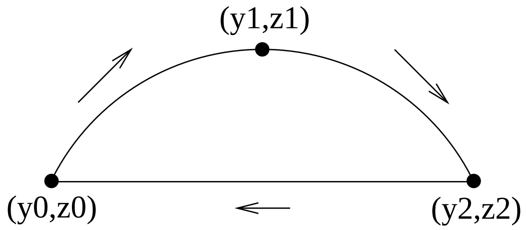

Figure 5: Leg trajectory

Building a walking gait

Let the leg trajectory just defined be the one of the leg 1. For the other three legs we must calculate the phase displacements in relation to the leg 1.

To be computationaly tractable we considered only a few of their values. We used 100 samples in a period of the trajectory.

Let d_{1i} be the phase displacement between the leg 1 and the leg i (with i=2,3,4). Then, d_{1i} must take full values between 0 and 99 that represent a fraction of the trajectory period.

We have, then, $100^3 = 10^6$ possible variations. We used an algorithm to test the stability of all the 100 robot configurations for all possible gaits. To do that we give different values to the duty factor that establish the fraction of full time cycle with the leg on the ground. We had arrived to following results:

- $dutyfactor = 80\% \rightarrow 1054$ gaits
- $dutyfactor = 76\% \rightarrow 58$ gaits
- $dutyfactor = 75\% \rightarrow 15$ gaits
- $dutyfactor = 74\% \rightarrow 1$ gait

For the case with a duty factor of 75% the phase displacement of the legs in relation to the leg 1 are around $T/2$, $T/4$ and $3T/4$ for the legs 2, 3 and 4, where T is the period of the gait (see table 1).

$\mathbf{d_{12}}$	$\mathbf{d_{13}}$	$\mathbf{d_{14}}$	$\mathbf{\Delta M(mm)}$
48	24	72	1.974174
49	24	73	3.912325
49	24	74	1.938593
49	25	74	1.938593
50	24	74	5.815778
50	24	75	1.938593
50	25	74	1.938593
50	25	75	3.842901
50	25	76	0.000000
50	26	75	0.000000
50	26	76	1.904721
51	24	75	3.912325
51	25	75	1.938593
51	25	76	1.938593
52	24	76	1.974174

Table 1: Stability margin for a duty factor of 75%

This results agree with some observations made in some animals (dogs and horses) whose slower gaits are similar to these ones. The walking gaits implemented in the Alegro platform showed the movements expected.

Stability margin

Another important study is the study of the stability margin. The stability margin of a gait is the minor of the minimum distances of the mass center to the border of the supporting polygon for all the robot configurations.

With this study we can test the sensibility of the gaits to disturbance of the mass center. A marginally static gait have a null stability margin so a small disturbance can change the position of the mass center to a position of instability.

We developed an algorithm which aim is to calculate the stability margin of the different walking gaits.

The results for the gaits derived for a 75% duty factor are shown in the table 1.

Conclusions

In this subsection we presented an algorithm to develop walking gaits and a method of analysis of the stability margin.

In the conditions of the walking gaits described we can say:

- The number of walking gaits grows with the duty factor;
- A duty factor smaller than 74% we can't derive any walking gait;
- All the walking gaits are periodic and regular. Some are also symmetric.

4. FUTURE WORK

In terms of future work there are many possibilities but our main piorities are:

1. Static gaits: we expect to improve the formal method of the exaustive search. There are several parameters that can be tuned in order to increase the performance of gaits derived by the method. We think that this model can give some very interesting results when applied to other robots. Another possible developement is an algorithm that with some parameters could choose the optimal gait between the gaits derived by the exaustive search.
2. Dynamic gaits: study the adaptation of gaits to variations in the ground slope.
3. Detection and avoidance of obstacles with the help of sensors (force, ultrasonic and infrared sensors) and its applications in dynamic gaits.
4. As a latter work we expect to do some research in navigation in natural terrain, using force and inercial sensors.

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